

An analysis of the oscillogram shows that distinct deviations of the light beam are observed at the same measurement point. This is evidently caused by the distinct resistance of the drops wetting the transducer.

In turn, the resistance depends on the diameter of the drops. Hence, the size of the drops incident on the transducer can be determined by means of the magnitude of the current.

Experiments conducted with a given different particle diameter, obtained by using a drop generator [4], afforded the possibility of constructing a calibration curve to determine the drop diameter as a function of the value of the current. Results of an analytical computation are compared in Fig. 3 with experimental results characterizing the depth of fluid particle penetration z_{\max} in the symmetric domain of the jet impingement zone or, in other words, the particle deceleration path in the axial direction, in order to analyze the hydrodynamics of the swirling jet impingement zone.

The results of an analytical investigation are verified completely satisfactorily by the experimental check.

The maximum value of the deceleration path fluctuated within the range 0.01-0.11 m for fluid drops of the size 100-1000 μ .

NOTATION

m , particle mass; γ_r , specific gravity of the gas; γ_p , specific gravity of the particle; F , middle section; c , frontal drag coefficient; Re , Reynolds number; ν , kinematic viscosity; d , particle diameter; W_{gl} , velocity at chamber exit; H , spacing between end faces; W_φ , W_r , W_z , tangential, radial, and axial gas velocities; τ , time; z_{\max} , maximum depth of particle penetration in the swirling impinging stream.

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BEHAVIOR OF POLYDISPERSE PARTICLE CLOUD IN GAS FLOW AT LOW REYNOLDS NUMBERS

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UDC 532.529.5

Expressions which describe the behavior of particles in a fluidized bed are obtained and studied. Comparative values are given for mass flow and energy of entrained particles as a function of relative concentration of particles of a different type.

A theoretical expression was obtained [1] with the help of a point-force approximation for the resistive force acting on a uniform polydisperse cloud of particles in the direction of gas flow at low Reynolds numbers.

The essence of the point-force approximation is the following: The perturbation introduced by a sphere in a flow at low Reynolds numbers is replaced by a point force applied to the center of the sphere. This force is assumed equal in magnitude but opposite in direction to the viscous force acting on a particle in the direction of flow.

All-Union Scientific-Research and Planning Institute of the Oil-Processing and Petrochemical Industries, Moscow. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 30, No. 2, pp. 235-239, February, 1976. Original article submitted September 11, 1974.

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The force acting on a particle of radius a_i in the direction of flow can then be represented as [1]

$$F_i = 6\pi\mu a_i f_i(\lambda) (v - w_i). \quad (1)$$

Equation (1) is a generalization of the well-known Stokes equation for flow around an isolated particle with λ defined by the expression

$$\lambda = \{6\pi m_2 + [36\pi^2 m_2^2 + 24\pi m_1 (1-3\rho/2)]^{1/2}\} / (2-3\rho),$$

where m_1 , m_2 , and m_3 are the first three moments of the particle-size distribution function $n(a)$,

$$m_n = \int n(a) a^n da,$$

and $\rho = 4\pi m_3/3$ is the volume concentration of particles.

For a system consisting of two sizes of particles, it was shown [2] that although the rate of entrainment of fine particles decreases as their relative concentration increases, their mass and energy flux increase as their relative concentration increases (for fixed overall volume concentration of the solid phase).

We consider a system consisting of particles of three sizes,

$$a_1 < a_2 < a_3, \quad \rho_1 < \rho_2, \quad \rho_3 < \rho_2,$$

where ρ_1 , ρ_2 , and ρ_3 are, respectively, the concentrations of the fine, medium, and coarse particles. Such a system corresponds better to the actual case.

In this case, m_1 and m_2 are given by the relations

$$m_1 = \frac{3}{4\pi a_1^2} \rho_1 + \frac{3}{4\pi a_2^2} \rho_2 + \frac{3}{4\pi a_3^2} \rho_3,$$

$$m_2 = \frac{3}{4\pi a_1} \rho_1 + \frac{3}{4\pi a_2} \rho_2 + \frac{3}{4\pi a_3} \rho_3,$$

where $\rho = \rho_1 + \rho_2 + \rho_3$.

Introducing the notation $a_2/a_1 = x > 1$, $a_2/a_3 = y < 1$, we obtain

$$\lambda a_2 = 4.5 [\rho + (x-1)\rho_1 + (y-1)\rho_3] / (2-3\rho) + \{20.25 [\rho + (x-1)\rho_1 + (y-1)\rho_3]^2 + 18 [\rho + (x^2-1)\rho_1 + (y^2-1)\rho_3] (1-3\rho/2)\}^{1/2} / (2-3\rho). \quad (*)$$

To determine the behavior of the particles, we write the sum of the forces acting on a particle. In the assumed model, gravity and the flow resistive force defined by Eq. (1) act on a particle. We then represent these equalities for type 1 and type 2 particles in the form

$$6\pi\mu a_i f_i(\lambda) (v - w_i) = -d_i^{4/3} \pi a_i^2 g,$$

$$f_i(\lambda) = 1 + \lambda a_i + 1/3 \lambda^2 a_i^2, \quad i = 1, 2. \quad (2)$$

Considering that the motion of the flow is directed oppositely to the vector for acceleration in free fall, we solve the system with respect to w_1 .

Assuming further that particles of radius a_2 are stationary (are balanced by the flow, $w_2 = 0$), we have the expression

$$w_1 = \left[1 - \frac{a_1^2 f_2(\lambda)}{a_2^2 f_1(\lambda)} \right] v = \frac{a_2^2 - a_1^2 + a_1 a_2 (a_2 - a_1) \lambda}{a_2^2 (1 + \lambda a_1 + 1/3 \lambda^2 a_1^2)} v, \quad (3)$$

for the steady-state velocity of the fine particles, where v is the flow velocity.

Introducing the notation $A = a_2^2 - a_1^2$, $B = a_1 a_2 (a_2 - a_1)$, $C = a_2^2$, $D = a_1 a_2^2$, and $E = a_1^2 a_2^2 / 3$, Eq. (3) can be written

$$w_1 = \frac{A + B\lambda}{C + D\lambda + E\lambda^2} v, \quad (4)$$

where $A, B, C, D, E > 0$.

Differentiating Eq. (4), we obtain

$$\frac{\partial w_1}{\partial \lambda} = - \frac{AD - BC + 2AE\lambda + BE\lambda^2}{(C + D\lambda + E\lambda^2)^2} v < 0,$$

since $AD - BC = a_1^2 a_2^2 (a_2 - a_1) > 0$.

Consequently, w_1 decreases as λ increases. As is clear from Eq. (*), however, λ increases with an increase in the relative concentration ρ_1 of the fine particles [for fixed concentrations ρ_3 (concentration of the coarse particles) and ρ (overall volume concentration of the solid phase)].

Thus, we find that w_1 falls with increase in the relative concentration of fine particles (and with a corresponding decrease in the concentration of medium particles). This conclusion follows qualitatively from consideration of Eq. (1). Thus, in the case where particles of type 2 are suspended, i.e., $w_2 = 0$, a gas velocity v of lower value is required for equalization of the weight of particles of type 2 when there is a rise in the coefficient of resistance (increase in λ). Consequently, the velocity of particles of type 1 must also be lower, since it is directly related to gas velocity.

When there is an increase in the relative concentration of coarse particles [for fixed concentrations ρ_1 (concentration of the fine particles) and ρ (the overall volume concentration of the solid phase)], λ falls.

Therefore, w_1 increases when there is an increase in the relative concentration ρ_3 of the coarse particles (with a corresponding decrease in the concentration of medium particles).

Proceeding in a similar manner with particles of types 2 and 3, we obtain an expression for the velocity of type 3 particles similar to Eq. (3),

$$w_3 = \left[1 - \frac{a_3^2 f_2(\lambda)}{a_2^2 f_3(\lambda)} \right] v = \frac{a_2^2 - a_3^2 + a_2 a_3 (a_2 - a_3) \lambda}{a_2^2 (1 + \lambda a_3 + 1/3 \lambda^2 a_3^2)} v.$$

Introducing the notation $A_1 = a_2^2 - a_3^2$, $B_1 = a_2 a_3 (a_2 - a_3)$, $C_1 = a_2^2$, $D_1 = a_2^2 a_3$, and $E_1 = a_2^2 a_3^2 / 3$, we rewrite the expression for the velocity w_3 in the form

$$w_3 = \frac{A_1 + B_1 \lambda}{C_1 + D_1 \lambda + E_1 \lambda^2} v.$$

From the conditions in our problem, $A_1, B_1 < 0$ and $C_1, D_1, E_1 > 0$.

We reach the obvious conclusion that the velocity of particles of the third kind is directly opposite to the direction of flow of the fluidizing agent.

Investigating the dependence of w_3 on λ , we obtain

$$\frac{\partial w_3}{\partial \lambda} = \frac{B_1 C_1 - A_1 D_1 - 2A_1 E_1 \lambda + B_1 E_1 \lambda^2}{(C_1 + D_1 \lambda + E_1 \lambda^2)^2} v.$$

The form of this relationship indicates that the magnitude of the velocity w_3 of the coarse particles can either increase or decrease when the relative concentration of fine particles increases depending on the relationship between the radii of the three types of particles.

For particles 10, 50, and 250 μ in size and an overall particle concentration of 0.5 in a bed, we calculate the mass flow J_i of particles in the i -th fraction and their energy flux Q_i :

$$J_i = \rho_i \frac{w_i}{v}, \quad Q_i = \rho_i \left(\frac{w_i}{v} \right)^3.$$

We consider three cases which are characterized by differing combinations of concentrations (Table 1).

The table makes it clear that the mass and energy flows of fine particles, increase with an increase in their relative concentration [for fixed concentrations ρ_3 (concentrations of coarse particles) and ρ (the overall volume concentration of the solid phase)], which agrees with the results of [2] for particles of two sizes. In addition, the table indicates that the mass flow and energy flow of fine particles also increase when there is a rise in the relative concentration of coarse particles.

TABLE 1. Mass Flow of Fine Particles from a Bed for a Fluidization Mode in Which Medium Particles Are Suspended

No.	ρ_1	ρ_2	ρ_3	J_1	Q_1
1	0,15	0,20	0,15	0,0585	0,00890
2	0,10	0,25	0,15	0,0446	0,00887
3	0,15	0,25	0,10	0,0372	0,00834

TABLE 2. Mass Flow of Fine and Medium Particles from a Bed for a Fluidization Mode in Which Coarse Particles Are Suspended

No.	ρ_1	ρ_2	ρ_3	w_1/v	w_2/v	J_1	J_2	Q_1	Q_2
1	0,15	0,20	0,15	0,46	0,105	0,0689	0,021	0,00146	0,000232
2	0,10	0,25	0,15	0,496	0,169	0,0496	0,042	0,0122	0,00121
3	0,15	0,25	0,10	0,444	0,11	0,0662	0,0275	0,0013	0,00033

In the general case where particles of type j are suspended, i.e., $w_j = 0$, the velocity of particles of type i is given by

$$w_i = \left[1 - \frac{a_i^2 f_j(\lambda)}{a_j^2 f_i(\lambda)} \right] v.$$

Calculated results for the quantities w_1/v , w_2/v , J_1 , J_2 , Q_1 , and Q_2 are given in Table 2 for the case where particles of type 3 are suspended.

As follows from the results given, entrainment of particles in the fine fraction decreases both with decrease in the concentration of particles in this fraction and also with decrease in the concentration of coarse particles. The total entrainment of fine and medium particles is held practically constant when there is a fixed concentration of coarse particles, and the total flow of fine and medium particles increases with a reduction in the concentration of coarse particles (with the overall concentration of the solid phase held fixed).

Thus, the relations presented and analyzed provide an opportunity to evaluate the effect of fractional composition on the amount of particle entrainment for various modes of fluidization.

NOTATION

μ , coefficient of flow viscosity; a , radius of solid particle; v , flow velocity; w , velocity of solid particles; ρ , volumetric fraction of solid phase in the medium; ρ_1, ρ_2, ρ_3 , concentrations of fine, medium, and coarse particles, respectively; a_1, a_2, a_3 , radii of fine, medium, and coarse particles; w_1 , velocity of fine particles; J_1 , mass flow of fine fraction; Q_1 , energy flow of fine fraction.

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